Space Dynamics

Core Topics:

Central force motion, determination of trajectory and orbital period in simple cases.

Special Topics:

Orbit transfer, in-plane and out-of-plane.

Space Dynamics Year Wise Analysis

Year	No of Questions	Total Marks
2018	1M : 3 2M : 1	5
2017	1M : 1	1
2016	2M : 1	2
2015	2M : 3	6
2014	1M : 1	1
2013	1M : 1	1
2012	1M:1	1
2011	2M:1	2
2010	1M:1 2M:1	3
2009	1M:1 2M:1	3
2008	1M:1 2M:3	7
2007	1M:1 2M:4	9

SPACE DYNAMICS

Force is the action of one physical body on another, either through direct contact or through a distance. Gravity is an example of force acting through a distance, as are magnetism and the force between charged particles. The gravitational force between two masses m1 and m2 having a distance r between their centres is

$$F_g = G\frac{m_1 m_2}{r^2}$$

This is **Newton's law of gravity**, in which G, the universal gravitational constant, has the value

$$G = 6.6742 \times 10^{-11}\ \frac{m^3}{Kg.s^2}$$

The force of a large mass (such as the earth) on a mass many orders of magnitude smaller (such as a person) is called weight, W. If the mass of the large object is M and that of the relatively tiny one is m, then the weight of the small body is

$$W = G\frac{Mm}{r^2}, \text{ But W = mg}$$

$$g = \frac{GM}{r^2}$$

Let g_0 represent the standard sea-level value of g, we get

$$g_0 = \frac{GM}{R_E^2}$$

Where, $g_0 = 9.81\frac{m}{s^2}$

Let z represent the distance above the earth's surface, so that r = R_E+ z,

$$g = \frac{g_0}{\left(1+\frac{z}{R_E}\right)^2}$$

1.1 GRAVITATIONAL PARAMETER (μ)

$$\mu = G(m_1 + m_2)$$

The units of μ are cubic kilometres per square second.

$$\mu = G(m_{Earth} + m_{Satelite})$$

$$m_{Earth} > m_{Satelite}$$

$$\mu_E = GM_{Earth} = g_0 R_E^2$$

$$\boldsymbol{\mu_E = 398600\ \frac{Km^3}{s^2}}$$

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1.5 THE ENERGY LAW

$$\boldsymbol{\frac{V^2}{2} - \frac{\mu}{r} = \varepsilon(constant)}$$

where ε is a constant. $\frac{V^2}{2}$ is the relative kinetic energy per unit mass. $(-\frac{\mu}{r})$ is the potential energy per unit mass of the body m_2 in the gravitational field of m_1. The total mechanical energy per unit mass ε is the sum of the kinetic and potential energies per unit mass. Above equation is a statement of the conservation of energy, namely, that the specific mechanical energy is the same at all points of the trajectory. Equation is also known as the **vis viva (“living force”) equation.** It is valid for any trajectory, including rectilinear ones.

For curvilinear trajectories, we can evaluate the constant ε at Periapsis (θ = 0),

$$\frac{V_p^2}{2} - \frac{\mu}{r_p} = \varepsilon = \varepsilon_p$$

where r_p and V_P are the position and speed at Periapsis. Since $V_r = 0$ at Periapsis, the only component of velocity is V_T, which means

$$V_T = V_p = \frac{h}{r_p}$$

Thus,

$$\epsilon = \frac{1}{2}\frac{h^2}{r_p^2} - \frac{\mu}{r_p}$$

Substituting the formula for Periapse radius in above expression,

$$r_P = \frac{h^2}{\mu}\frac{1}{1+e}$$

We get,

$$\boldsymbol{\epsilon = -\frac{1}{2}\frac{\mu^2}{h^2}(1-e^2)}$$

Clearly, the orbital energy is not an independent orbital parameter.

NOTE: The energy E of a spacecraft of mass m is obtained from the specific energy ε by the formula

$$E = m\epsilon$$

1.6 CIRCULAR ORBITS (e = 0)

Setting e = 0 in the orbital equation,

$$r = \frac{h^2}{\mu}\frac{1}{1+eCos\theta}$$

We get,

$$\boldsymbol{r = \frac{h^2}{\mu}}$$

That is, r = constant, which means the orbit of m_2 around m_1 is a circle. Since the radial velocity V_r is zero, it follows that V = V_T so that the angular momentum formula h = rV_T becomes simply h = rV for a circular orbit. Substituting in above expression, we get,

$$r = \frac{r^2V^2}{\mu}$$

$$\boldsymbol{V_{circular} = \sqrt{\frac{\mu}{r}}}$$

The **time T** required for one orbit is known as the period.

$$T = \frac{circumference}{speed} = \frac{2\pi r}{\sqrt{\frac{\mu}{r}}}$$

$$\boldsymbol{T_{circular} = \frac{2\pi}{\sqrt{\mu}}r^{3/2}}$$

The **specific energy** of a circular orbit is found by setting e = 0 in,

$$\epsilon = -\frac{1}{2}\frac{\mu^2}{h^2}(1-e^2)$$

We get,

$$\boldsymbol{\epsilon = -\frac{1}{2}\frac{\mu^2}{h^2} = -\frac{1}{2}\frac{\mu}{r}}$$

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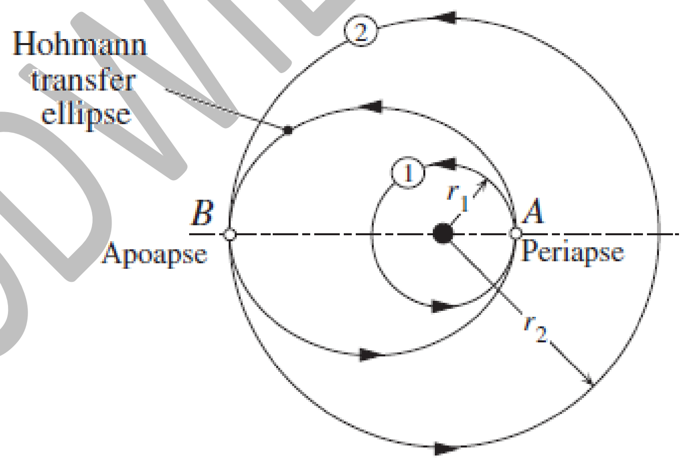
1.11 HOHMANN TRANSFER

- The Hohmann transfer is the most energy-efficient two-impulse manoeuvre for transferring between two coplanar circular orbits sharing a common focus.
- The Hohmann transfer is an elliptical orbit tangent to both circles on its apse line.
- The Periapsis and apoapsis of the transfer ellipse are the radii of the inner and outer circles, respectively.
- Only one-half of the ellipses is flown during the maneuver, which can occur in either direction, from the inner to the outer circle, or vice versa.



- Energy of an orbit depends only on its semi-major axis a, for an ellipse, the specific energy is negative,

$$\boldsymbol{\epsilon = -\frac{1}{2}\frac{\mu}{a}}$$

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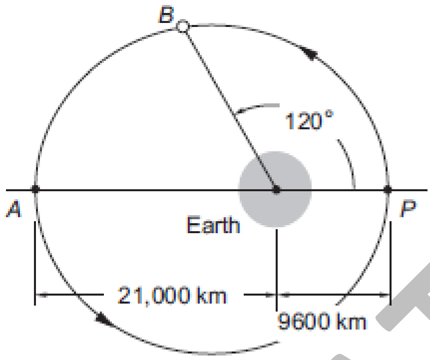
Example1: A geocentric elliptical orbit has a perigee radius of 9600 km and an apogee radius of 21,000 km, as shown in. Calculate the time period of the orbit.



Solution:

The eccentricity is readily obtained from the perigee and apogee radii

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{21000 - 9600}{21000 + 9600} = 0.37255$$

We find the angular momentum using the orbit equation,

$$9600 = \frac{h^2}{398600} \frac{1}{1 + 0.37255 \cos 0}$$

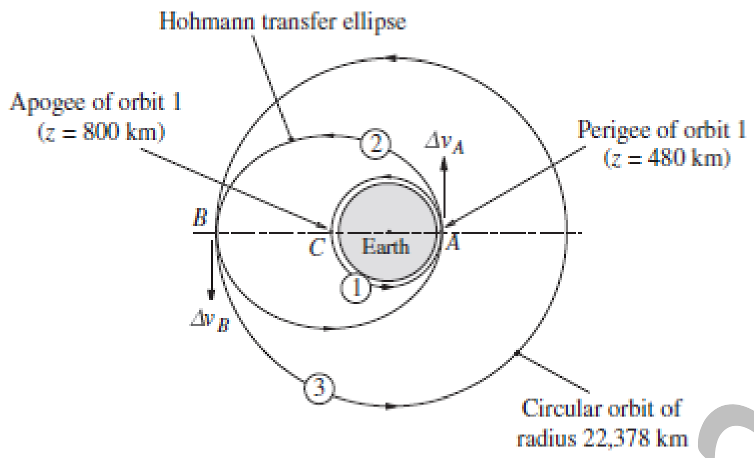
$$\therefore \mathbf{h = 72472\ km^2/s}$$

With h and e, the period of the orbit is obtained from

$$T = \frac{2\pi}{\mu^2}\left(\frac{h}{\sqrt{1-e^2}}\right)^3 = \frac{2\pi}{398600^2}\left(\frac{72472}{\sqrt{1-0.37255^2}}\right)^3 = \mathbf{18834\ sec}$$

Example2: A 2000-kg spacecraft is in a 480 x 800 km earth orbit (orbit 1 in Figure). Find

(a) The Δv required at perigee A to place the spacecraft in a 480 x 16,000 km transfer ellipse (orbit 2).
(b) The Δv (apogee kick) required at B of the transfer orbit to establish a circular orbit of 16,000 km altitude(orbit3).
(c) The total required propellant if the specific impulse is 300 s.



Solution:

Since we know the perigee and apogee of all three of the orbits, let us first use the equation to calculate their angular momentum.

Orbit 1: r_p = 6378 + 480 = 6858 km, r_a = 6378 + 800 = 7178 km

$$\therefore h_1 = \sqrt{2 \text{ x } 398600}\sqrt{\frac{7178 \text{ x } 6858}{7178 + 6858}} = 52876.5 \text{ km}^2/\text{s}$$

Orbit 2: r_p = 6378 + 480 = 6858 km, r_a = 6378 + 16,000 = 22,378 km

$$\therefore h_2 = \sqrt{2 \text{ x } 398600}\sqrt{\frac{22378 \text{ x } 6858}{22378 + 6858}} = 64689.5 \text{ km}^2/\text{s}$$

Orbit 3: $r_a = r_p$ = 22,378 km

$$\therefore h_2 = \sqrt{398600 \text{ x } 22378} = 94445.1 \text{ km}^2/\text{s}$$

(a) The speed on orbit 1 at point A is

$$v_A)1 = \frac{h_1}{r_A} = \frac{52876}{6858} = 7.71019 \text{ km/s}$$

The speed on orbit 2 at point A is

$$v_A)2 = \frac{h_2}{r_A} = \frac{64689.5}{6858} = 9.43271 \text{ km/s}$$

Therefore, the delta-v required at point A is

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1.12 PLANE CHANGE MANEUVERS

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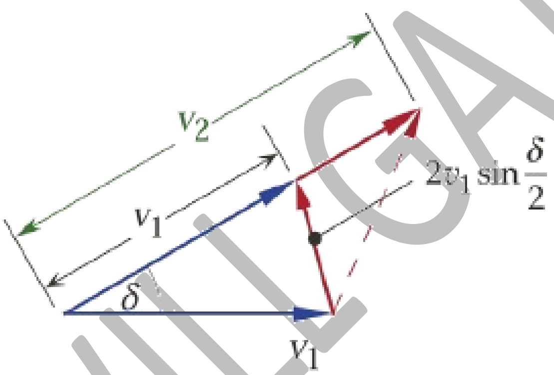
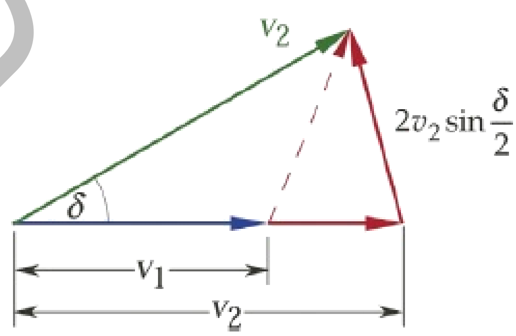
- ***Case B:*** **Rotate the velocity vector and then change its magnitude**



$$\Delta v_{II} = 2v_1 sin\frac{\delta}{2} + |v_2 - v_1|$$

- ***Case C:*** **Change the speed first, and then rotate the velocity vector**



$$\Delta v_{III} = |v_2 - v_1| + 2v_2 sin\frac{\delta}{2}$$

NOTE: Since the sum of the lengths of any two sides of a triangle must be greater than the length of the third side, it is evident from above figures that both Δv_{II} and Δv_{III} are greater thanΔv_I. **It follows that plane change accompanied by speed change is the most efficient of the above three maneuvers.**

GATE and Additional Questions

Q1. In an elliptic orbit around any planet, the location at which a spacecraft has the maximum angular velocity is **[GATE 2018]**

(a) Apoapsis
(b) Periapsis
(c) c appoint at $\pm 45^0$ from Periapsis
(d) appoint at $\pm 90^0$ from apoapsis

Ans: (b)

The angular momentum depends only on the **azimuthal (perpendicular or transverse) component of the relative velocity and is constant for an orbit.**

$$\boldsymbol{h = rV_T} = \textbf{constant}$$

The larger the value of r, i.e. farthest position (apoapsis) of the mass, the velocity is minimum and the smaller the value of r, i.e. closest position (Periapsis) of the mass, the velocity is maximum.

Q.2 The radius of the earth is 6.37×10^6m and the acceleration due to gravity at its surface is $9.81 m/s^2$. A satellite is in circular orbit at a height of 35.9×10^6m above the earth's surface. This orbit is inclined at 10.5 degrees to the equator. The velocity change needed to make the orbit equatorial is: **[GATE 2007]**

(a) 561 m/s at 84.75 degrees to the initial direction
(b) 561 m/s at 95.25 degrees to the initial direction
(c) 281 m/s at 84.75 degrees to the initial direction
(d) 281 m/s at 95.25 degrees to the initial direction

Ans: (b) 561 m/s at 95.25 degrees to the initial direction

Given: $R_E = 6.37 \times 10^6$m,
$g = 9.81 m/s^2$,
$\delta = 10.5^0$

$$r = 6.37 \times 10^6 + 0.63 \times 10^6 = 35.9 \times 10^6 m = 42.27 \times 10^6$$

$$V_1 = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{42.27 \times 106}} = 3.07\ km/s$$

$$\therefore \Delta V = 2V \sin\frac{\delta}{2} = \mathbf{561\ m/s}$$

$$\delta + 2\alpha = 180^0$$

$$\Theta = 180 - \alpha$$

$$\delta + 2(180 - \Theta) = 180^0$$

$$\therefore 10.5 + 360 - 2\Theta = 180^0$$

$$\therefore \theta = \mathbf{95.25^0}$$

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EXERCISE

Q1. A satellite in Earth orbit has a semi-major axis of 6,700 km and an eccentricity of 0.01. Calculate the satellite's altitude at both perigee and apogee.

Ans: 254.9 km and 388.9 km

Q2. A satellite is in an orbit with a semi-major axis of 7,500 km and an eccentricity of 0.1. Calculate the time it takes to move from a position 30 degrees past perigee to 90 degrees past perigee.

Ans: 968.4 s

Q3. A spacecraft launched from Earth has a burnout velocity of 11,500 m/s at an altitude of 200 km. What is the hyperbolic excess velocity?

Ans: 3,325 m/s

Q4. A satellite in Earth orbit passes through its perigee point at an altitude of 200 km above the Earth's surface and at a velocity of 7,850 m/s. Calculate the apogee altitude of the satellite in km.[Take Radius of Earth = 6378.14 km]

Ans: 427

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